Performance Evaluation of a Multi-Part JIT Production System

Mitsutoshi KOJIMA* and Kenichi NAKASHIMA**

*Department of Civil Engineering and Systems Management, Nagoya Institute of Technology
**Department of Technology Management, Osaka Institute of Technology

Abstract: Under stochastic demand and deterministic processing times, we discussed a single-part JIT production system with the production-ordering and supplier Kanbans and derived the stationary distributions of the backlogged demand. In this paper, we extend the system to a multi-part JIT production system, deterministic processing times and withdrawals with lead time. These conditions are more realistic than the previous papers. We formulate the system and show that a shortage of some parts degrades the performance of the system in full production case. A part with the minimum average number of supplier Kanbans which has great influence on the performance is especially focused. Moreover, the occurrence probability of backlogged demand is calculated and the importance of increasing the number of Kanbans of the parts with the minimum average number of supplier Kanbans is shown by using simulation.

Keywords JIT, Kanban Control, Performance Evaluation, Supply Chain Management.

1. Introduction

Supply chain management (SCM) plays an important role in various kinds of business environments. Taylor [1] insists that the apparently simple concept such as low inventories with deliveries supplied just-in-time (JIT) for production process has far-reaching effects internal to the firm and externally throughout the supply chain. A JIT production system is a well-known successful model to design an effective supply chain management. It is applied to various business, such as not only the assembly of a car but also the supply of electronic components or medical supplies in Japan. It is based on two concepts; one is JIT and the other is Autonomation that means defect free. These concepts include institutional aspects to manage the systems at the view point of manufacturing resources such as materials, labors and information. In the JIT production systems, the kanban is used as a tool to coordinate the both information and material flows[2].

Deleersnyder et al.[3] have investigated effects of factors such as the number of kanbans, the machine reliability and the demand variability on the performance of a JIT production system with only the production-ordering kanban using a discrete time Markov chain. Under stochastic demand and deterministic processing times, Ohno, et al. [4] discussed a single-stage JIT production system with the production-ordering and supplier Kanbans and derived a probability generating function of the stationary distributions of the backlogged demand. In this paper, we expand the system to a multi-stage JIT production system with two kinds of Kanbans under stochastic demand, deterministic processing times and withdrawals with lead time. These conditions are more realistic than those of Mitra and Mitrani [5][6], Tayur[7], Kirkavak and Dinçer[8] and Berkley [9].


We formulate a multi-part JIT production system with two kinds of Kanbans under stochastic demand, deterministic processing times and withdrawals with lead time. The effect of the multi-part environment on the performance of the system is analyzed in full production case. A part with the minimum average number of supplier Kanbans which has great influence on the performance is especially focused. Moreover, the occurrence probability of backlogged demand is calculated by using simulation and the influence of increasing the number of supplier Kanbans of the parts is evaluated.

2. Multi-part JIT Production System

We expand the JIT production system with supplier and production-ordering Kanbans [4] into a multi-part production system, which is shown in Fig. 1. A
product consists of two or more parts. For simplicity, take the constant delivery cycle as one period. In this system, the demand is stochastic and the processing time is deterministic. The following notation is used:

- \( j_{\text{max}} \): the number of the parts,
- \( j \): the index of the part (\( 1 \leq j \leq j_{\text{max}} \)),
- \( L_j \): the delivery lead time of part \( j \),
- \( M \): the number of production-ordering Kanbans,
- \( N_j \): the number of supplier Kanbans of part \( j \),
- \( C \): the production capacity,
- \( D(k) \): the demand in period \( k \),
- \( B(k) \): the backlogged demand at the beginning of period \( k \),
- \( I_j(k) \): the inventory level of part \( j \) at the beginning of period \( k \),
- \( J(k) \): the number of the production-ordering Kanban in the production-ordering Kanban post at the beginning of period \( k \),
- \( P(k) \): the production quantity in period.

The order of each part consumed in period \( k = 1, 2, \cdots \) is transmitted to the supplier of the part at the beginning of period \( (k + 1) \), and they are delivered at the beginning of period \( (k + L_j + 1) \). It is assumed that the demand of the product in each period is independent and identically distributed with distribution \( \{ \text{Pr} \{ D(k) = d \} = p_d, \ d = 0, 1, \cdots, D_{\text{max}} \} \) and mean \( D \), the excess demands of the products are backlogged and the container capacity is equal to one.

Since \( L_j \) is the delivery lead time of the supplier of part \( j \), \( N_j \) denotes the number of supplier Kanbans of part \( j \) and the number of supplier Kanbans transmitted to the supplier at the beginning of periods \( k \) is \( P(k-1) \), it holds that for \( k = 1, 2, 3, \cdots \),

\[
N_j = I_j(k) + \sum_{m \in \mathbb{N}_j} P(m), \ 1 \leq j \leq j_{\text{max}} \quad (1)
\]

where \( P(0), P(-1), \cdots, P(-L_j + 1) \) are given.

The inventory level of part \( j \) at the beginning of period \( (k + 1) \) is changed from that of periods \( k \) by the difference between the delivery quantity from the supplier at the beginning of period \( k \) and the consumed quantity of part \( j \) in period \( k \), it holds that

\[
I_j(k+1) = I_j(k) + P(k-L_j) - P(k), \ 1 \leq j \leq j_{\text{max}}. \quad (2)
\]

In the JIT production system, the production quantity is determined by the minimum among the inventory level of each part, the production-order quantity
and the production capacity. That is,

\[ P(k) = \min(I_1(k), \ldots, I_{j_{\text{max}}}(k), J(k), C). \]  

(3)

The backlogged demand occurs at the beginning of period \((k+1)\) if the sum of the backlogged demand at the beginning of period \(k\) and the demand from the customers in period \(k\) exceeds the sum of the production quantity and the inventory level of produced parts, \(M-J(k)\). Therefore,

\[ B(k+1) = [B(k) + J(k) + D(k) - P(k) - M]^+, \]  

(4)

where \([x]^+ = \max(0, x)\).

Since the number of production-ordering Kanbans at the production-ordering Kanban post at the beginning of period \((k+1)\) is the minimum between \(M\) and the total backlogged demand at the beginning of period \((k+1)\),

\[ J(k+1) = \min(M, B(k) + J(k) + D(k) - P(k)). \]  

(5)

3. The Effect of Multi-part Environment

By using equations (1)-(5) in Section 2, the performance of the multi-part JIT production system is analyzed at the time of making full Kanban circulation. The production quantity and inventory level in the JIT system are calculated under the conditions that demand is very larger than the supply quantity of parts as follows:

\[ D(k) = 100, k = 1, 2, \ldots, \]  

\[ j_{\text{max}} = 2, (L_1, L_2) = (2, 4), M = 100, (N_1, N_2) = (15, 21), \]  

\[ C = 100, \quad B(1) = 100, \]  

\[ (I_1(1), I_2(1)) = (15, 21) \]  

and \(J(1) = 100\). By setting the initial backlogged demand and the customers demand in each period as very large values, as long as there are both parts 1 and 2, a product is always produced.

Moreover, the performances of the single-part (only part 1) JIT production system and the single-part (only part 2) JIT production system are also given for comparison.

Table 1. Performance at the time of full Kanban circulation

<table>
<thead>
<tr>
<th>Case-1: the multi-part JIT production system</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1(k))</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>(I_2(k))</td>
<td>21</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>15</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>(\Sigma(N))</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Case-2: the single-part (only part 1) JIT production system

\[ \begin{array}{ccccccccccccccc}
\hline
k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
I_1(k) & 15 & 0 & 0 & 15 & 9 & 9 & 6 & 0 & 9 & 9 & 6 & 0 & 9 & 9 & 6 \\
J_1(k) & 0 & 15 & 0 & 15 & 0 & 15 & 0 & 15 & 0 & 15 & 0 & 15 & 0 & 15 & 0 \\
P_1(k) & 15 & 15 & 15 & 30 & 30 & 45 & 45 & 45 & 60 & 60 & 60 & 75 & 75 & 75 & 75 \\
\hline
\end{array} \]

Case-3: the single-part (only part 2) JIT production system

\[ \begin{array}{ccccccccccccccc}
\hline
k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
I_2(k) & 21 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 \\
P_2(k) & 21 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 \\
\hline
\end{array} \]

When producing with single-part as Case-2 and Case-3 in Table 1, it turns out that the production is performed periodically in a cycle of the delivery lead time \(L_j + 1\). The production quantity is equal to the number of supplier Kanbans of single-part. In the multi-part JIT production system, the accumulative production quantity \(\sum_{j=1}^k P(k)\) does not exceed the accumulative production quantity in single-part cases. Especially in periods 7, 8, 11, 12, and 13, it is the inferior value rather than any in the cases of single-part JIT production system. This shows that the performance is degraded because of run out of parts. Therefore, when the number of supplier Kanbans is the same, it is suggested that the occurrence probability of backlog demand in the multi-part JIT production system is higher than one in the single-part JIT production system.

From the results in the case of Table 1 and other various cases, the minimum average number of supplier Kanbans of part \(j_{\text{ave}}\), which is defined by the following equation, mainly determines the performance of the multi-part JIT production system,

\[ j_{\text{ave}} = \arg \min \left\{ \frac{N_1}{L_1+1}, \frac{N_2}{L_2+1}, \ldots, \frac{N_j}{L_j+1} \right\}. \]  

(6)

In the case of Table 1, \(j_{\text{ave}} = 2\). Moreover, from equation (1) the production quantity in the multi-part JIT production system during \(L_{j_{\text{ave}}}+1\) periods is less than \(N_{j_{\text{ave}}}\). Therefore, in order to improve the supply capacity of the parts of the JIT production system, it is necessary to increase the number of supplier Kanbans of part \(j_{\text{ave}}\) preferentially.

4. Simulation Result

The performance of the multi-part JIT production system is evaluated by using simulation. A numerical simulation was performed under the follow-
Kanbans which has great influence on the performance of the production system. If you want to improve the supply capacity of the parts of the JIT production system, it is necessary to increase the number of supplier Kanbans of part $j_{ave}$ with preferentially. Moreover, the occurrence probability of backlogged demand is calculated and the importance of increasing the number of supplier Kanbans of the parts with the minimum average number of supplier Kanbans is shown by using simulation. From the simulation result, when the minimum average number of supplier Kanbans is increased, it will become possible to raise the performance to the backlogged demand efficiently. On the other hand, increasing the number of Kanbans leads to an increase of the holding cost of parts. Therefore, optimization of the number of Kanbans will be needed in consideration of the balance of backlogged demand cost of products and holding cost.

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References

5. Conclusion
We have formulated the multi-part JIT system and showed that run out of parts degrades the performance of the system in full production case. A part $j_{ave}$ with the minimum average number of supplier Kanbans which has great influence on the performance of the production system. If you want to improve the supply capacity of the parts of the JIT production system, it is necessary to increase the number of supplier Kanbans of part $j_{ave}$ with preferentially. Moreover, the occurrence probability of backlogged demand is calculated and the importance of increasing the number of supplier Kanbans of the parts with the minimum average number of supplier Kanbans is shown by using simulation. From the simulation result, when the minimum average number of supplier Kanbans is increased, it will become possible to raise the performance to the backlogged demand efficiently. On the other hand, increasing the number of Kanbans leads to an increase of the holding cost of parts. Therefore, optimization of the number of Kanbans will be needed in consideration of the balance of backlogged demand cost of products and holding cost.

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References
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Mitsutoshi KOJIMA
He received M.S. and Ph. D. degrees in engineering from Nagoya Institute of Technology, Japan, in 1991 and 2006, respectively. From 1991 to 1994, he was a computer engineer with Fujitsu Limited, Japan. From 1994-2007, he is a research associate in Nagoya Institute of Technology. Since 2007, he has been an associate professor with the department of civil engineering and systems management, Nagoya Institute of Technology. His research interests include production planning, inventory control and profitability analysis.

Kenichi NAKASHIMA
He is an Associate Professor in the Department of Industrial Management at Osaka Institute of Technology in Osaka, Japan. He received his BE, MS and PhD in Industrial Engineering from Nagoya Institute of Technology, Nagoya, Japan, in 1990, 1992 and 1995, respectively. From 1995 to 1996, he was a Research Associate in the Department of Industrial Management, Osaka Institute of Technology. From 1996 to 2000, he was an Assistant Professor in the Department of Industrial Management, Osaka Institute of Technology. He was also a visiting assistant professor at MIT in 1998. Dr. Nakashima received the Research Award for young researchers from Japan Industrial Management Association in 1997. He has served as a member of technical committees of international conferences. His research interests include dynamic programming, Markov decision processes, production management and management information systems. His papers have appeared in International Journal of Production Research, International Journal of Production Economics and IEEE Transactions on Systems, Man and Cybernetics (Part A).